Enhancing Decision Topology Assessment in Engineering Design

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Abstract

Implications of decision analysis (DA) on engineering design are important and well-documented. However, widespread adoption has not occurred. To that end, the authors recently proposed *decision topologies* (DT) as a visual method for representing decision situations and proved that they are entirely consistent with normative decision analysis. This paper addresses the practical issue of assessing the DTs of a designer using their responses. As in classical DA, this step is critical to encoding the DA's preferences so that further analysis and mathematical optimization can be performed on the correct set of preferences. We show how multi-attribute DTs can be directly assessed from DM responses. Furthermore, we show that preferences under uncertainty can be trivially incorporated and that topologies can be constructed using single attribute topologies similarly to multi-linear functions in utility analysis. This incremental construction simplifies the process of topology construction. The methods are demonstrated using a design decision making problem of a welded beam.

1. Introduction

The notion that engineering design is a decision making process is well accepted and documented (see Howard (1989), Thurston (1991) and Hazelrigg (1998). However, despite all possible benefits, widespread use of decision making tools is impeded by the complexity of these tools to the average designer. Most practicing engineers are not trained in decision analysis or utility theory and as a result, they implement "best practices" when it comes to making decisions. Even when an expert attempts to elicit the DM's preferences, errors are introduced, affecting the quality of decisions to be made. These errors may result from problems associated with stated and revealed preferences (Train, 2003). It is also possible that the facilitator (expert helping the DM to make the decision) overwhelms the DM with lottery questions to assess utility functions, particularly when the DM is not entirely comfortable giving responses to lottery questions. The authors have recently proposed an alternative approach based on Decision Topologies, which alleviates these issues with classical decision analysis and also offers many significant advantages (Pandey and Mourelatos, 2013). They also showed that DTs are entirely consistent with decision analysis at the limit. In this paper, we make theoretical advances in DTs by focusing on their assessment directly. We show that DTs can be constructed from single attribute topologies similarly to multi-linear functions in utility analysis. This makes DT assessment simple.

The paper is arranged as follows. Section 2 discusses decision topologies and Section 3 presents theoretical results in DT assessment. Section 4 describes a case study involving a welded beam demonstrating the topology assessment and the decision making process using the topology. Section 5 concludes and discusses directions for future work.

2. Decision Topologies

What is a decision topology?

A decision topology is a block diagram similar to the reliability block diagrams in reliability engineering (Figure 1) where a system is operational if its block diagram representation has a continuous path from one side of the diagram (input) to another (output). The decision topology extends this notion. If there are no paths from one end to the other, the DT is assigned a score of zero. Otherwise, all paths from one side of the diagram to the other are counted providing an overall score for the DT. This score is a positive linear transformation of the decision maker's utility function, a claim that has been substantiated in Pandey and Mourelatos (2013). We provide a sketch of the proof later for continuity.

We use a simple example to show how DTs work. Consider a car-purchasing decision where the attributes are fuel economy M in miles per gallon (mpg), acceleration A in seconds for 0 to 60 mph, roominess R in cu. ft., price P in dollars, luxury L (acceptable/unacceptable) and color C (acceptable/unacceptable). Some attributes such as luxury and color, are not easy to define mathematically because they are subjective. Figure 1 shows an example DT.

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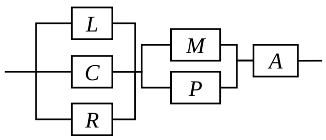


Figure 1. Reliability block diagram (decision topology) in a hypothetical vehicle buying decision.

The given DT can be interpreted using simple reliability engineering principles (Kapur and Lamberson, 1977). The algorithm for identifying topologies similar to the one above, and scoring them is based on our recent work towards identifying system topologies with limited information (Pandey and Mourelatos, 2012b) and evaluating a reliability block diagram using linear algebra operations (Pandey et al., 2012).

2.1 Proof of consistency with decision analysis

Many types of Multi-Attribute Utility Functions (MAUF) exist (e.g., multi-linear, linear, Cobb-Douglas, etc.) allowing for different tradeoff behaviors. We have shown how decision topologies are consistent with decision analysis by showing that the behavior of any continuously differentiable utility function which is monotonic in attributes can be modeled by decision topologies. This is *in addition* to binary attributes, which can be easily incorporated.

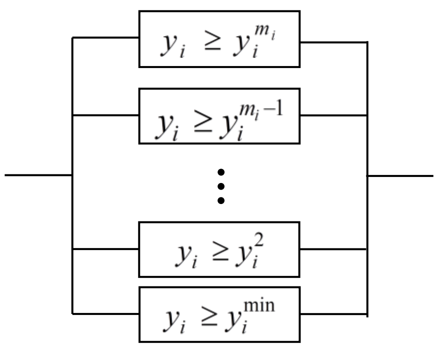


Figure 2. A decision topology for a single attribute (superscript is not an exponent).

Let us start with the single attribute case, where the objective is to find the score corresponding to an attribute level. We can represent the decision topology associated with an attribute visually as shown in Figure 2. The decision topology consists of blocks where each block tests the binary condition that the attribute level is greater than a partition y_i^j of the attribute. If this is true, the block is considered active. The *raw* score for an attribute level y_i is equal to the number of paths from right to left (or left to right) through the active blocks. For example, if $y_i \geq y_i^{\max}$ all blocks will be active because all inequalities are satisfied. Similarly, if $y_i < y_i^{\min}$ none of the blocks will be active. For $y_i^{\min} < y_i < y_i^{\max}$, a number of blocks less than m_i will be active. Figure 3 pictorially shows the score from a DT as the attribute level is steadily increased and the blocks become progressively active.

Since utility functions are conventionally constrained between 0 and 1, we can divide the score of an attribute level by m_i to obtain a *normalized* score which we refer to as score *S*. It may seem that the score will be linear in attribute level (akin to a linear utility function) but this is not true. The way the score varies with respect to an attribute depends on the chosen partitioning for the attribute. A non-uniform partitioning can be used to draw the topology. If the density of partitions around a particular value of y_i is proportional to the derivative of the utility function at that value of y_i, the normalized score will be equal to the utility value (Figure 3). This is guaranteed if $m_i \rightarrow \infty$ since the score approaches the Riemann sum under the utility density function (derivative of the utility function). A rigorous proof is given in Pandey and Mourelatos (2013).

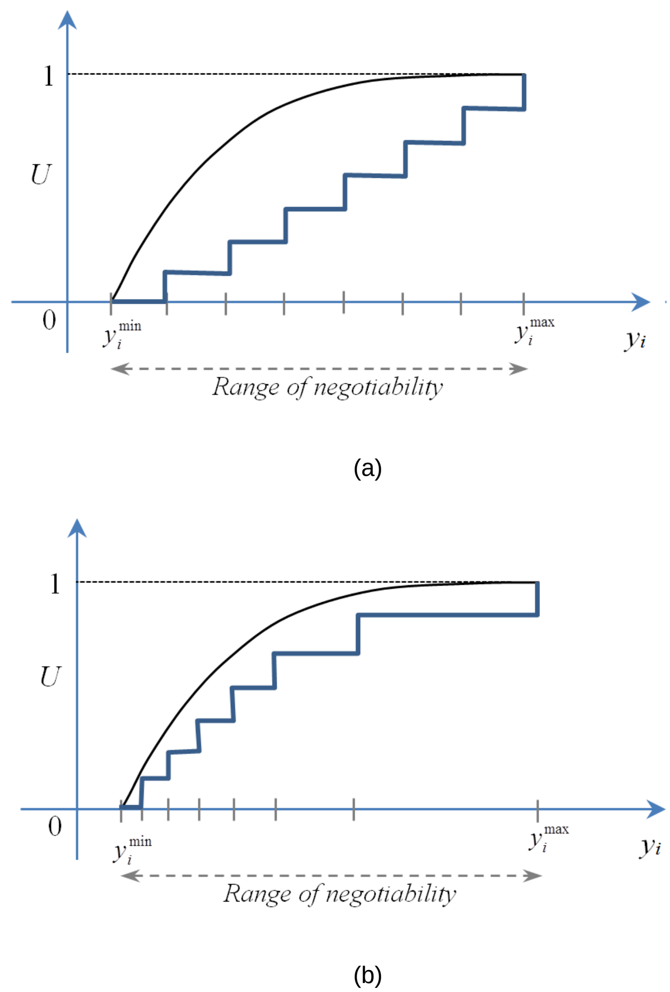


Figure 3. Comparison of a utility function with a decision topology score when the first partitioning of the attribute is uniform (a) or proportional to the local value of the derivative of the utility function (b).

Tradeoffs

The multi-attribute case is less straightforward compared to the single attribute case. For this reason, we motivate the reader by first showing that tradeoffs are also represented in a topology when multiple attributes are present. Consider the mileage and price block of Figure 1. Assume that the two attributes have three levels simplistically designated as low, medium and high. A tradeoff structure can be introduced as in Figure 4 which expands upon the price-mpg block of Figure 1. Based on Figure 4, the decision maker will buy a vehicle that meets their low price requirements even if it gives low mpg (top path). Similarly, a vehicle that provides average mpg at average initial price is also acceptable (middle path). Finally, a costly vehicle will be acceptable if it provides high mpg (bottom path).

Now we prove that the score determined from the topology is the same as that from a utility function. As before, we assume that the multi-attribute utility function is continuously differentiable and increasing in individual attributes. Also, each attribute is partitioned within its range of negotiability. The partition of each attribute results in a block of the form $\{y_i \geq y_i^j\}$ in the topology. Since we have m_i blocks for an attribute *i*, the total number of block combinations is equal to $\prod_{i=1}^{n} m_i$. In this case, the most intuitive way to draw the decision topology is to use a tuple of blocks in each row of the decision topology as shown in Figure 5. The decision topology will then have $\prod_{i=1}^{n} m_i$ rows in parallel. The score from such a topology mimics the multi-attribute utility function and is monotonic in each attribute, as expected. Moreover, $S(y_1 \geq y_1^{\max},\ldots,y_n \geq y_n^{\max}) = 1$ and $S(y_1 = y_1^{\min},\ldots,y_n = y_n^{\min}) = 0$.

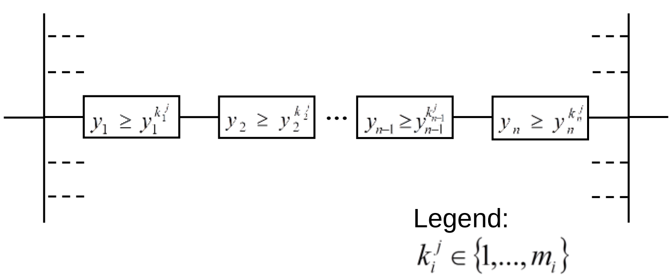


Figure 4. An example block representing tradeoff between price and mpg for the vehicle buying example.

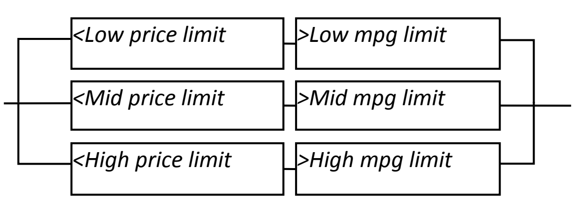


Figure 5. Decision topology for multi-attribute case (only one row is shown).

If the CDF analogy is used, the proof that DTs are consistent with DA can be derived using the arguments for the single attribute case. We have

$$S(y_1^t,\ldots,y_n^t) = \int_{y_1^{\min}}^{y_1^t} \cdots \int_{y_n^{\min}}^{y_n^t} u_{y_1,\ldots,y_n}(y_1,\ldots,y_n)\,dy_1 \ldots dy_n = U(y_1^t,\ldots,y_n^t) \quad (1)$$

where $u_{y_1,\ldots,y_n}(y_1,\ldots,y_n)$ is the joint utility density function as defined in Abbas (2009) which is analogous to probability density functions. The analogy however, is not applicable to utility functions which are not *n*-increasing (i.e., having a negative mixed derivative; see Nelsen, 1999). Because such utility functions are common, we provide below a general proof that is valid for all continuously differentiable utility functions.

Assume that we start with the point $\{y_1^{\min},\ldots,y_n^{\min}\}$ where the utility function and the score $S(y_1^{\min},\ldots,y_n^{\min})$ are both zero. If the attribute vector is perturbed by a small amount $\{dy_1,\ldots,dy_n\}$ in any general direction that keeps the attributes in the range of negotiability, the new utility can be approximated by

$$U(y_1^{\min}+dy_1,\ldots,y_n^{\min}+dy_n) \approx \frac{\partial U}{\partial y_1}dy_1 + \ldots + \frac{\partial U}{\partial y_n}dy_n . \quad (2)$$

Similarly to the single attribute case, if the first partitioning at every point along any attribute is proportional to the partial derivative along that attribute, we have

$$S(y_1^{\min}+dy_1,\ldots,y_n^{\min}+dy_n) \approx U(y_1^{\min}+dy_1,\ldots,y_n^{\min}+dy_n). \quad (3)$$

As the partitioning gets finer, the approximation of Equation (3) keeps improving. As we move on a path from $\{y_1^{\min},\ldots,y_n^{\min}\}$ towards any point $\{y_1^t,\ldots,y_n^t\}$, the decision topology makes successive linear approximations of the utility function, and

$$S(y_1^t,\ldots,y_n^t) \approx U(y_1^t,\ldots,y_n^t). \quad (4)$$

Equation (4) proves that if the partitioning of the attribute space is sufficiently fine, and the distribution of the partitions at any point along any direction is proportional to the partial derivative of the utility function along that direction, the decision topologies will provide the same score with the value of the utility function.

In practical applications, the decision topology of Figure 5, with rows corresponding to each block tuple, can be modified to fine tune the tradeoff behavior. Additionally, the topology is not necessarily constrained to have $\prod_{i=1}^{n} m_i$ rows in parallel, as described earlier. The algorithm we have developed, called Evolutionary System Topology Approximation (ESTA) to evolve decision topologies from limited data can find creative arrangements of blocks which can concisely represent the tradeoff information (Pandey and Mourelatos, 2012a). Furthermore, the partitioning does not even have to be fine for one to approximate a utility function well. We substantiate these claims in the results section.

Some clarifications

Is a Decision Topology simply a decision tree?

The answer to this question is no. Decision trees are a visual representation of the entire decision-making situation. The main function of decision topologies is to replace the utility function. Decision trees quickly become intractable as more nodes are added while decision topologies give a good picture of the decision situation without becoming intractable. In decision trees, a decision at a node is made by comparing the expected utility from the alternatives downstream the node. In that respect, a decision topology can be used in conjunction with a decision tree, where utility values are replaced with decision topology scores.

What about uncertainty?

Uncertainty can be incorporated in the decision topology by definition. Similarly to calculating the expectation of a utility function in decision analysis, we can calculate the expectation of the score provided by the decision topology. If uncertainty must be incorporated directly, we can evolve topologies using tests (see below) that involve uncertainty.

3. Assessing Decision Topologies

We have proven that DTs can approximate utility functions. Therefore if the utility function is available, one can fit a DT to it using our proposed algorithm. This will help visualize the decision problem once a decision is to be made. On the other hand, it is possible that the utility function is not available and the DTs are to be assessed directly. It is also possible that Single Attribute utilities, and equivalently, Decision Topologies (SADTs) are available and one must find the Multi-Attribute Decision Topology (MADT). We first show the method for assessing the MADT directly and then show how it can be constructed from SADTs.

Theoretical Considerations

If risk attitude is not to be modeled, as in cases where the choices are deterministic, the utility functions and the value functions are the same (up to a linear transformation). In this case, we propose having the decision maker rank different multi-attribute tuples according to their tradeoff preferences. ESTA can then be used to approximate the topology so that the Spearman's rank correlation between the ranking provided by the DM and that provided by a candidate topology is high. The output of the algorithm is the topology that provides the highest rank correlation with the decision maker's ranking. While ESTA has provable convergence at the limit, we may not run the simulation long enough because of time constraints. Results are still very good as our recent work has shown (Pandey and Mourelatos, 2012b).

In case the risk attitude must be modeled, we must include uncertain choices in the questions asked to the DM. These are called lottery questions in the DA literature. The DM is asked a combination of deterministic and lottery questions. Table 1 for example, shows a small subset of ranking questions.

Table 1: Sample ranking questions that can be used to assess MADTs directly.

Rank these alternatives in the order of your preference:	
A	$\mathbf{y_1} = \{y_1^1, ..., y_n^1\}$
B	60% chance of $\mathbf{y_{2P}} = \{y_1^{2P}, ..., y_n^{2P}\}$ and 40% chance of $\mathbf{y_{2Q}} = \{y_1^{2Q}, ..., y_n^{2Q}\}$
C	$\mathbf{y_3} = \{y_1^3, ..., y_n^3\}$
D	10% chance of $\mathbf{y_{4P}} = \{y_1^{4P}, ..., y_n^{4P}\}$ and 90% chance of $\mathbf{y_{4Q}} = \{y_1^{4Q}, ..., y_n^{4Q}\}$
E	$\mathbf{y_5} = \{y_1^5, ..., y_n^5\}$

Notice the method does not require a departure from the ESTA method that we use for deterministic choices. The relative number of uncertain and deterministic options in the alternatives is a matter of judgment and the amount of effort the assessor and DM are willing to put in. As in any utility assessment procedure, the outcomes are a direct function of the effort invested. The guarantee of our method is that the assessment will monotonically improve with the number of questions asked. The assessor should obviously ensure that the questions are all very different from each other so all the regions within the ranges of negotiability are properly modeled.

We now come to the issue of assessing MADTs from SADTs. It is well known that using the multi-linear expression of Keeney and Raiffa (1994) requires that the attributes be preferentially and utility independent. These conditions can somewhat be relaxed, i.e., other functional forms are possible to get MAUFs from SAUFs directly, as shown in Abbas (2009). Abbas discusses utility copulas and makes a case for modeling MAUFs as joint distributions using copulas. Functionally, many utility functions have the same properties as joint distributions and therefore copulas, as a result of Sklar's theorem, can be a powerful tool to model them. It is also directly evident that SAUFs can be combined using a *utility copula* to get a multi-attribute utility copula (MAUF). Certain functional forms where the mixed derivative of the MAUF is negative, a condition not true for probability distributions, can also be modeled using utility copulas. Furthermore, the grounding condition where probability copulas are zero when one of the variables is at its lowest level, is also not an impediment because, again as

Abbas notes, there are utility copulas that do not need to satisfy the grounding condition. As such, we do not find any reasons to not build MADTs directly from SADTs.

4. Case Study

We use a welded beam design example adapted from Deb, Pratap and Moitra (2000) with bound constraints $g_5(\mathbf{x})$ and $g_6(\mathbf{x})$ added to the original problem. The problem involves simultaneous minimization of the cost (*C* in dollars) and the deflection (*D* in inches) of the weld. We do not discuss the results of the optimization here because our focus is simply to find a topology consistent with the utility function. However, we use the information that the two attributes *C* and *D* can be feasibly realized in the ranges of [10, 260] and [0.001, 0.05].

We first define an exponential utility function over each attribute as

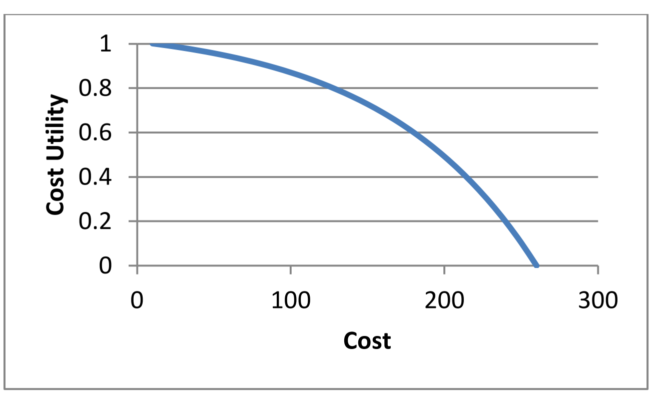
$$U_1(C) = \left(1 - e^{-\frac{260-C}{100}}\right) \tag{5}$$

$$U_2(D) = \left(1 - e^{-\frac{0.05-D}{0.02}}\right) \tag{6}$$

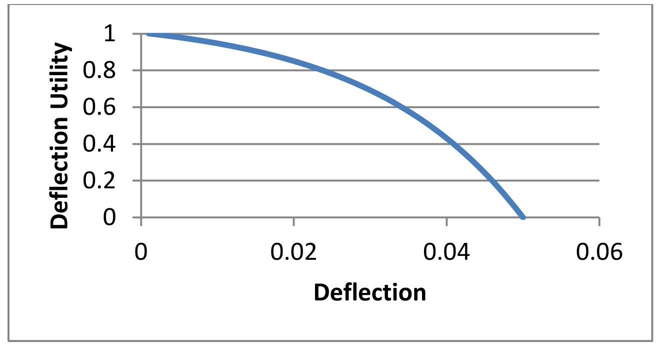
and then combine the utilities into a multi-attribute function using the following multi-linear form

$$U(U_1, U_2) = 0.7U_1 + 0.5U_2 - 0.2U_1U_2 . \tag{7}$$

We assume that while these utility functions are unknown to the assessor, they do model the decision maker's preferences correctly. Notice that *U* does not satisfy the grounding condition, and it also has a negative mixed derivative. We will generate survey questions using these utility functions. However, using the proof shown before, we get the SADTs directly from the derivative characteristics of the utility functions in Figure 6. Recall that for the SADTs to work, all we must do is partition the domain proportionally to the derivative of the utility functions. Table 2 shows this partitioning.



(a)



(b)

Figure 6. Utility functions associated with the two objectives. The utilities are decreasing w.r.t. the attributes without affecting the applicability of the method.

Table 2. Partitioning of the ranges of negotiability of the two attributes using the derivative information of Figure 6.

Cost ($)	Deflection (in)
20	0.005
120	0.025
190	0.038
230	0.045
250	0.049

Table 3 provides the training set. The DM is asked to rank the alternatives in the order of desirability. Of the total of 15 alternatives, 5 are probabilistic. Notice that the total number of alternatives is significantly less than that used in our previous work. This is possible because we have already extracted useful information from the SADTs (Table 2).

Table 3. Ranking provided by the DM for the 15 alternatives.

		Cost ($)	Deflection (in)		Cost ($)	Deflection (in)	Ranking
Deterministic outcomes		234.71	0.0088				9
		218.69	0.0446				13
		247.67	0.0486				15
		226.93	0.0468				14
		158.32	0.0149				3
		148.45	0.0138				2
		204.12	0.0024				7
		76.60	0.0208				1
		235.91	0.0144				10
		124.81	0.0316				5
	Probability	Outcome 1		Probability	Outcome 2		
Probabilistic outcomes	0.15	86.25	0.0488	0.85	193.03	0.0066	6
	0.65	238.29	0.0218	0.35	109.56	0.0484	11
	0.2	157.75	0.0092	0.8	137.72	0.0439	8
	0.8	228.69	0.0218	0.2	174.44	0.0473	12
	0.3	139.40	0.0407	0.7	99.52	0.0261	4

Next the ESTA algorithm is run to create a topology that will provide the same rankings (or close to it) as in Table 3. ESTA was run with a population size of 350 and a probability of mutation of 0.1. The total run-time on a 2.13 GHz dual-core Intel Xeon machine was less than a minute. Figure 7 shows the topology generated by ESTA. This topology gives a Spearman's rank correlation of 0.97, which is excellent.

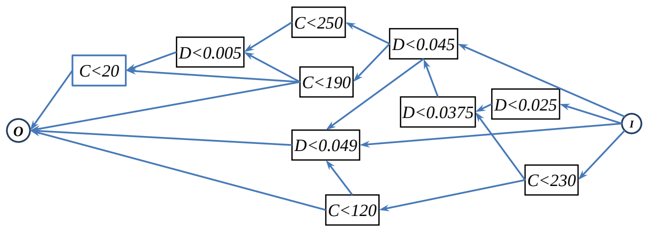


Figure 7. MADT generated using the responses provided by the decision maker.

Table 4 shows the score and the ranking provided by the best MADT found by the ESTA algorithm. The rankings are an excellent match, with a very high rank correlation coefficient which guarantees that the decisions made using the MADT, even under uncertainty, will be the same as those made by the decision maker himself.

Table 4. Score and ranking by best MADT found by ESTA.

Alternative	MADT score	MADT rank	DM rank
1	3	12	9
2	2	13	13
3	1	15	15
4	1	14	14
5	7	3	3
6	7	2	2
7	4	6	7
8	9	1	1
9	3	11	10
10	5	5	5
11	3.85	7	6
12	3	10	11
13	3.8	8	8
14	3.4	9	12
15	5.8	4	4
Spearman's rank correlation		0.97	

Summary/Conclusions

In this paper, we proposed methods to assess multi-attribute decision topologies (MADTs) for making design decisions. The authors had recently shown that MADTs are visual, theoretically sound replacements of utility functions and are consistent with decision analysis at the limit. This paper addressed three major challenges in assessing MADTs. We showed that MADTs can be directly assessed using rankings provided by the decision maker using an evolutionary method we have previously proposed. We then showed that preferences under uncertainty can be trivially incorporated in MADTs by adding probabilistic alternatives in the ranking questions. Finally, we showed that if single attribute DTs are available, we can extract information from it so that the assessment of MADTs is simplified and the DM only ranks a few alternatives to obtain the complete MADT. We also discussed the theoretical basis behind the assessment method we proposed.

We used a case study of a welded beam design with two attributes. With only 15 alternatives (10 deterministic and 5 probabilistic) we were able to get a MADT with a Spearman's rank correlation of 0.97 with the DM's rankings. The high value of the Spearman's rank correlation validates our method for assessing MADTs from SADTs even under uncertainty. We believe that MADTs are a significant contribution to the state-of-the-art in engineering design decision making.

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Appendix

The mathematical formulation of the welded beam example is given below.

Minimize

$$\begin{cases} C(\mathbf{x}) = 1.10471h^2l + 0.04811tb(14+l) \\ D(\mathbf{x}) = \dfrac{2.1952}{t^3b} \end{cases}$$

subject to $g_i(\mathbf{x}) \le 0,\ i = 1,\ldots,5$

where $\mathbf{x} = (h,l,t,b)^T$, and

$$g_1(\mathbf{x}) = \tau(\mathbf{x}) - 13600,\ g_2(\mathbf{x}) = \sigma(\mathbf{x}) - 30000,\ g_3(\mathbf{x}) = h - b$$

$$g_4(\mathbf{x}) = 6000 - P_c(\mathbf{x}),\ g_5(\mathbf{x}) = \mathbf{x} - (10,10,10,5)^T \text{ and } g_6(\mathbf{x}) = (0.125,0.1,0,0)^T - \mathbf{x}$$

In the above expressions,

$$\tau(\mathbf{x}) = \sqrt{\tau_1^2 + \tau_2^2 + \frac{l\tau_1\tau_2}{\sqrt{0.25\left(l^2 + (h+t)^2\right)}}}$$

$$\tau_1 = \frac{6000}{\sqrt{2}hl}$$

$$\tau_2 = \frac{6000(14+0.5l)\sqrt{0.25\left(l^2+(h+t)^2\right)}}{2\left(0.707hl\left(\dfrac{l^2}{12} + 0.25(h+t)^2\right)\right)}$$

$\sigma(\mathbf{x}) = \dfrac{504000}{t^2b}$ and $P_C(\mathbf{x}) = 64746.022(1 - 0.0282346t)tb^3$